Mini-Coil Design

Michael von Ortenberg

Fukui University, Okayama University,
Humboldt University at Berlin

presented at

Mini Coil Workshop

IMR, Tohoku University, November 24,2006

PARAMETERS of the Mini-Coils constructed:

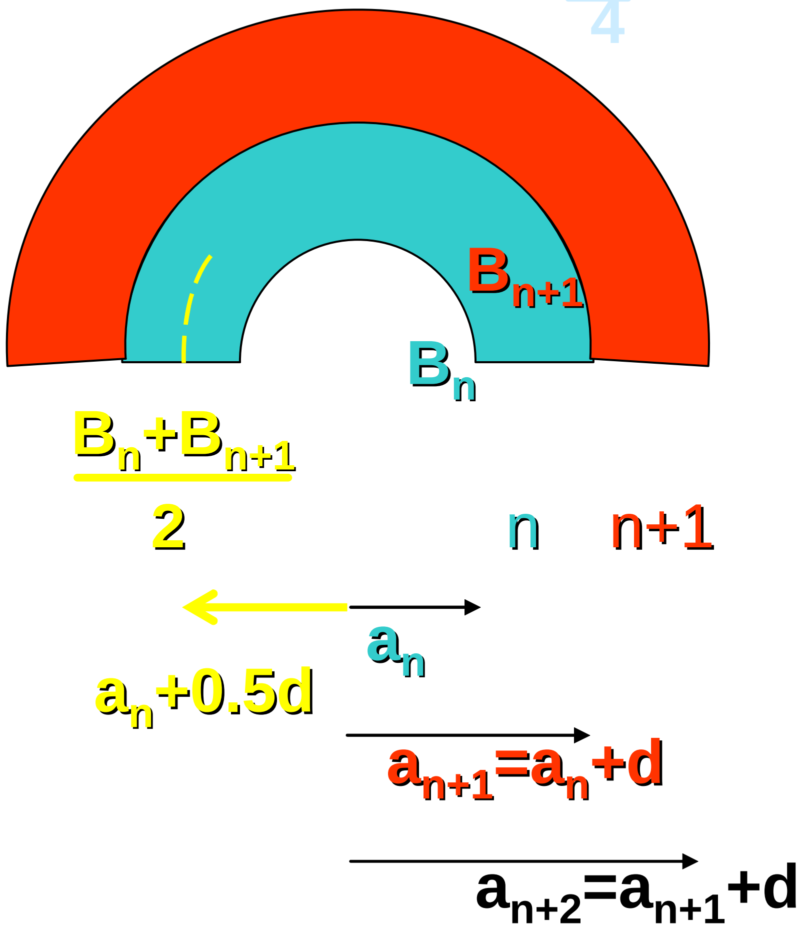
Coil	A	B	C	D	E
Bore (mm)	3/2.6	3/2.6	3/2.6	3/2.6	3/2.6
Wire (mm)	0.5 Cu/Ag	0.5 Cu/Ag	0.5 Cu/Ag	0.5 Cu/Ag	2x0.5 Cu/Ag
Number of layers	12	16	14	12	12
Total height (mm)	20	20	20	15	20
R_{300} (Ohm) With steel flanges	1.017	1.76	1.304	0.747	0.285
R_{300} (Ohm) Without steel flanges	-	-	-	0.728	0.285
R_{77} (Ohm) with steel flanges	0.257	0.371	-	-	-
L_{300} (μH) without steel flanges	-	-	-	138,7	-
L_{300} (μH) with steel flanges	292	758	549.7	182.45	86
B/U_{coil} (T/V)	2.98/100	1.30/102	2.82/101	3.64/101	3.20/103
B_{max} (T)/U_{coil}	47.78/1997	41.06/1997	46.29/1997	51.85/1806	51.72/1805
$\tau/2 = (t_{Bmax} - t_{B0})$ (msec) at 77 K	0.74	1.03	0.886	0.564	0.414
Operation	crowbar	crowbar	crowbar	crowbar	crowbar

Schematic of Construction:

Schematic of Calculation:

$$B_{n+1}=B_n-\Delta Bn(a_n)$$

$$\sigma_n=\frac{(B_n+B_{n+1})}{2}*I*\frac{(a_n+0.5d)}{\frac{\pi d^2}{4}}$$



Math-CAD Program:

Program SpulOkayama-Sendai.mcd for the computation of MINI COILS in OKAYAMA   for Cu/Ag-wire with breaking tension 0.78 G-Pa
Version 26.10.06 based on SpulNeu.mcd of 7.11.02

$i := \sqrt{-1}$

Parameters:

$a1 := 1.5$	***[mm] inner radius of the coil***
$la := 20$	***[mm] total height of coil***
$I0 := 1.78$	***[kA] current in the wire***
$Bm := 48$	***[T] maximum field of coil***
$d := 0.5$	***[mm] diameter of wire***
$\rho d := 9.96$	***[gr/cm-3] density Cu***
$NL := 12$	***: number of windings***

Capacitor banks #1 and #2:

$C1 := 7.2\cdot 10^{-3}$	***Farad***	$U1 := 5000$	***Volt***	***90 kJ***
$C2 := 0.96\cdot 10^{-3}$	***Farad***	$U2 := 2000$	***Volt***	***4 kJ***

Current density in wire at maximum current:

$j := \frac{I0}{0.25\cdot\pi\cdot d^2}$ $j = 9.065$ ***[kA/mm^2] in wire***

$je := \frac{I0}{d^2}$ $je = 7.12$ ***:[kA/mm^2]   effective homogeneous current density***

Fieldfunctions in the ho mogeneous coil:
Field [T] in the coil axis at height z [mm] above the center:
a [mm] inner radius of the one-layer coil, la [mm] total height of the coil, I current in kA

$$B(z,a,la,I) := \frac{1.26\cdot I}{2\cdot d}\cdot\left[\frac{0.5-\frac{z}{la}}{\sqrt{\left(0.5-\frac{z}{la}\right)^2+\left(\frac{a}{la}\right)^2}}+\frac{0.5+\frac{z}{la}}{\sqrt{\left(0.5+\frac{z}{la}\right)^2+\left(\frac{a}{la}\right)^2}}\right]$$

: for effective homogneous current density I0/(d^2)

Field [T] at position y [mm] for z=0 inside or outside of the coil:

$$BE(y,a,la,I) := \frac{1.26\cdot I}{2\cdot\pi\cdot d}\cdot\int_0^{\pi}\frac{1-\left(\frac{y}{a}\right)\cdot\cos(\phi)}{\left[1+\left(\frac{y}{a}\right)^2-2\cdot\left(\frac{y}{a}\right)\cdot\cos(\phi)\right]\cdot\sqrt{0.25+\left(\frac{y}{la}\right)^2-1\cdot\left(\frac{y}{la}\right)\cdot\cos(\phi)+\left(\frac{a}{la}\right)^2}}\,d\phi$$

Remark: BE(y) has a pronounced step at the position y=a and reduces to 0 for larger y.

Fieldstrength BF(0,y,z) inside and outside the coil:
y-component:

$$BFy(y,z,a,la,I) := \frac{-1.26\cdot I}{2\cdot\pi\cdot d}\cdot\int_0^{\pi}\frac{\cos(\phi)}{\left[\left(\frac{la}{2\cdot a}+\frac{z}{a}\right)^2+1-2\cdot\frac{y}{a}\cdot\cos(\phi)+\left(\frac{y}{a}\right)^2\right]^{0.5}} \dots + \frac{-\cos(\phi)}{\left[\left(\frac{la}{2\cdot a}-\frac{z}{a}\right)^2+1-2\cdot\frac{y}{a}\cdot\cos(\phi)+\left(\frac{y}{a}\right)^2\right]^{0.5}}\,d\phi$$

z-component:

$$BFz(y,z,a,la,I) := \frac{-1.26\cdot I}{2\cdot\pi\cdot d}\cdot\int_0^{\pi}\frac{\left(\frac{y}{a}\cdot\cos(\phi)-1\right)\cdot\left(\frac{la}{2\cdot a}-\frac{z}{a}\right)}{\left[1-2\cdot\frac{y}{a}\cdot\cos(\phi)+\left(\frac{y}{a}\right)^2\right]\cdot\left[\left(\frac{la}{2\cdot a}-\frac{z}{a}\right)^2+1-2\cdot\frac{y}{a}\cdot\cos(\phi)+\left(\frac{y}{a}\right)^2\right]^{0.5}} \dots + \frac{\left(\frac{y}{a}\cdot\cos(\phi)-1\right)\cdot\left(\frac{la}{2\cdot a}+\frac{z}{a}\right)}{\left[1-2\cdot\frac{y}{a}\cdot\cos(\phi)+\left(\frac{y}{a}\right)^2\right]\cdot\left[\left(\frac{la}{2\cdot a}+\frac{z}{a}\right)^2+1-2\cdot\frac{y}{a}\cdot\cos(\phi)+\left(\frac{y}{a}\right)^2\right]^{0.5}}\,d\phi$$

Computation of the tension in element (d x d) in homogeneous coil enclosing wire of diameter d:

a: inner radius of the winding considered in [mm]
d: wire diameter in [mm]
B: Magnetfield produced by the winding abover the considered winding in [T],
j: Current density within the considered winding in [kA/mm^2]

$Bb_1 := Bm$ $\qquad a_1 := 1.5$

$n := 2 .. NL + 1$

$a_n := a_{n-1} + d$

$Bb_n := Bb_{n-1} - B\left(0, a_{n-1}, la, I0\right)$

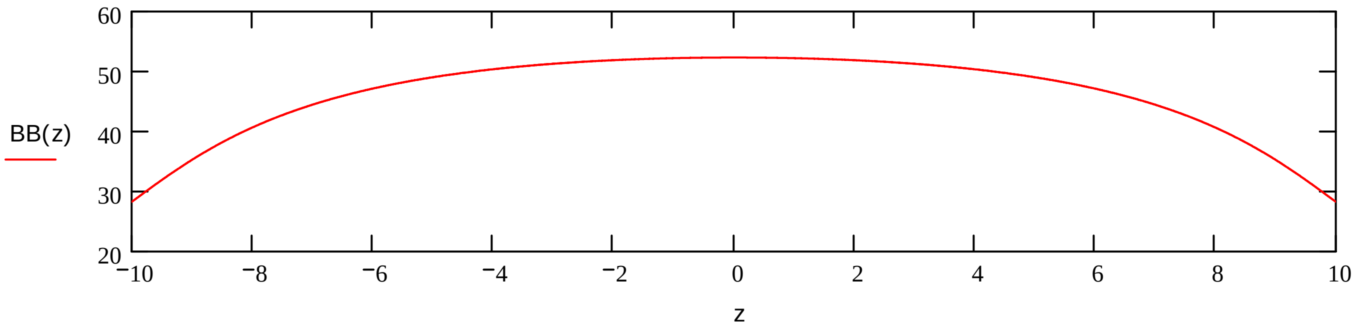
$\sigma_{n-1} := \frac{\left(Bb_{n-1}+Bb_n\right)}{2}\cdot I0\cdot\frac{\left(a_{n-1}+\frac{d}{2}\right)}{\left(d^2\right)}\cdot 10^{-3}$ ***: G-Pa in square element (d x d) in contrast to wire cross section πd^2/4***

$$\sigma_{NL+1} := \frac{Bb_{NL+1}}{2} \cdot I0 \cdot \frac{\left(a_{NL+1} + \frac{d}{2}\right)}{\left(d^2\right)} \cdot 10^{-3} \qquad \sigma_{NL} = 0.033$$

Total diameter of coil: $Dtotal := 2 \cdot \left(a_{NL} + d\right)$ $\qquad Dtotal = 15$ $\qquad$ ***: [mm]***

Calculation of the z-dependence of the field in the center axis:

$$BB(z) := \sum_{n=1}^{NL+1} B\left(z, a_n + 0.5 \cdot d, la, I0\right)$$



$$\frac{BB(2)}{BB(0)} = 0.991$$

Hence within +/- 2 mm around center field inhomgeneity very small!

Total number of windings: $NL = 12$

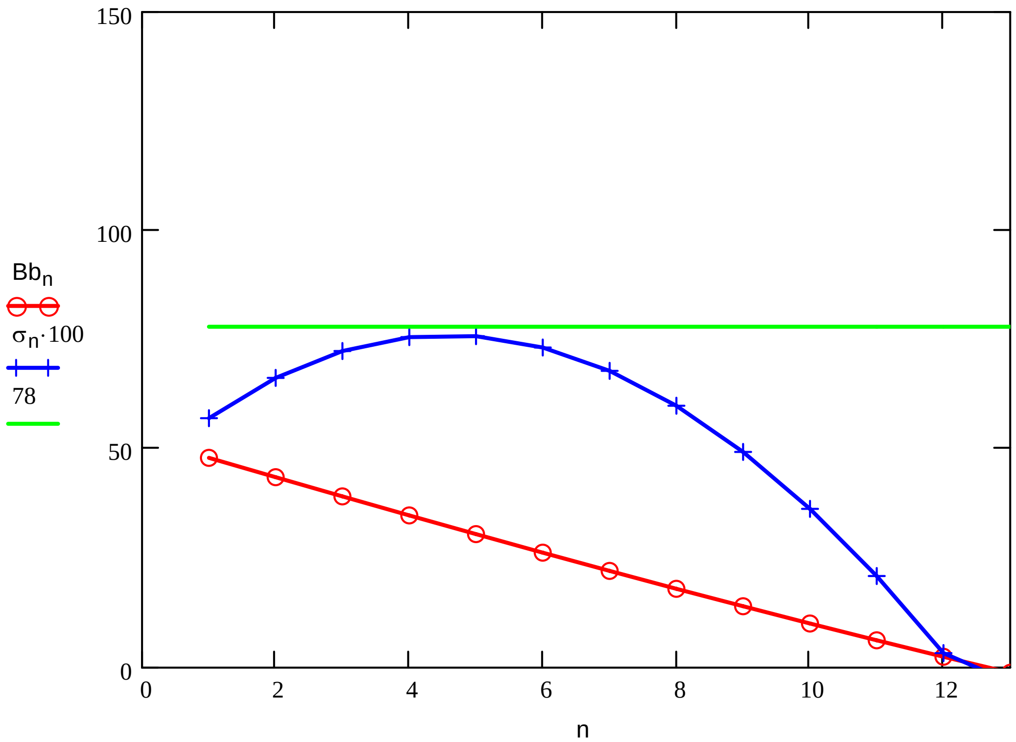
a: inner winding radius in mm $\qquad$ ***Bb: magnetic field inside winding in T***

σ; strain in wire of winding in G-Pa

a =	Bb =	σ =
0	0	0
1.5	48	0.57
2	43.564	0.663
2.5	[illegible]	[illegible]
3	34.814	0.756
3.5	30.517	0.758
4	26.284	0.732
4.5	22.119	0.679
5	18.028	0.599
5.5	14.016	0.493
6	10.086	0.363
6.5	6.24	0.21
7	2.479	0.033
7.5	−1.196	−0.033

$$'\cdot,^{3} := \sum_{liter = 1}^{NL} \left(a_{liter} + \frac{d}{2}\right)\cdot 2\cdot\pi\cdot\frac{la}{d}$$ $$'\cdot,^{3} = 1.357 \times 10^{4}$$ ***: mm total wire length***

$n := 1 .. NL + 1$



$del := 0.25001$ ***: [mm]***

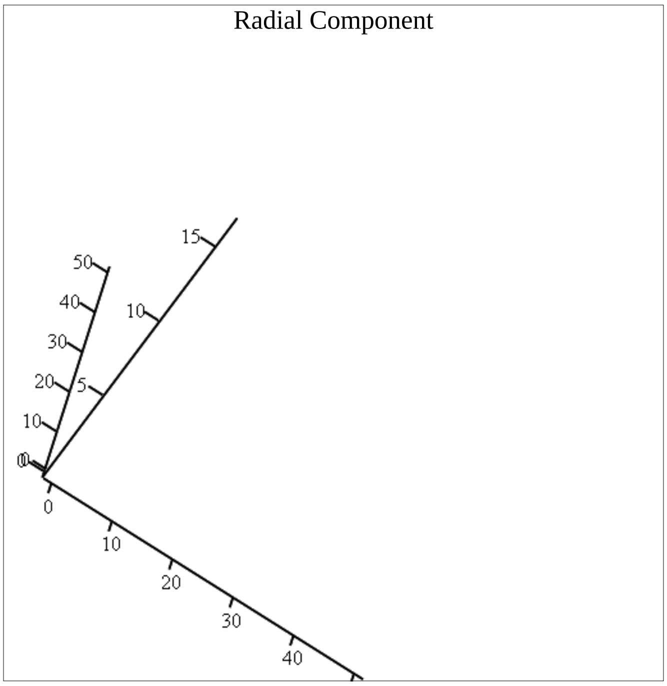
$n := 0 .. 50$

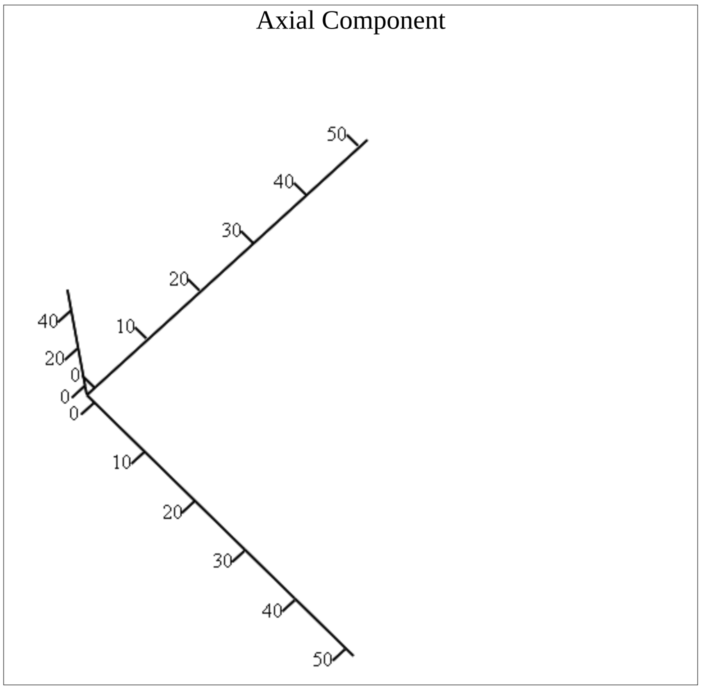
$m := 0 .. 50$

$$My_{n,m} := \sum_{liter = 1}^{NL} BFy\left(del\cdot n, del\cdot m, a_{liter}, la, I0\right)$$

$$Mz_{n,m} := \sum_{liter = 1}^{NL} BFz\left(del\cdot n, del\cdot m, a_{liter}, la, I0\right)$$

$M_{n,m} := My_{n,m} + i \cdot Mz_{n,m}$





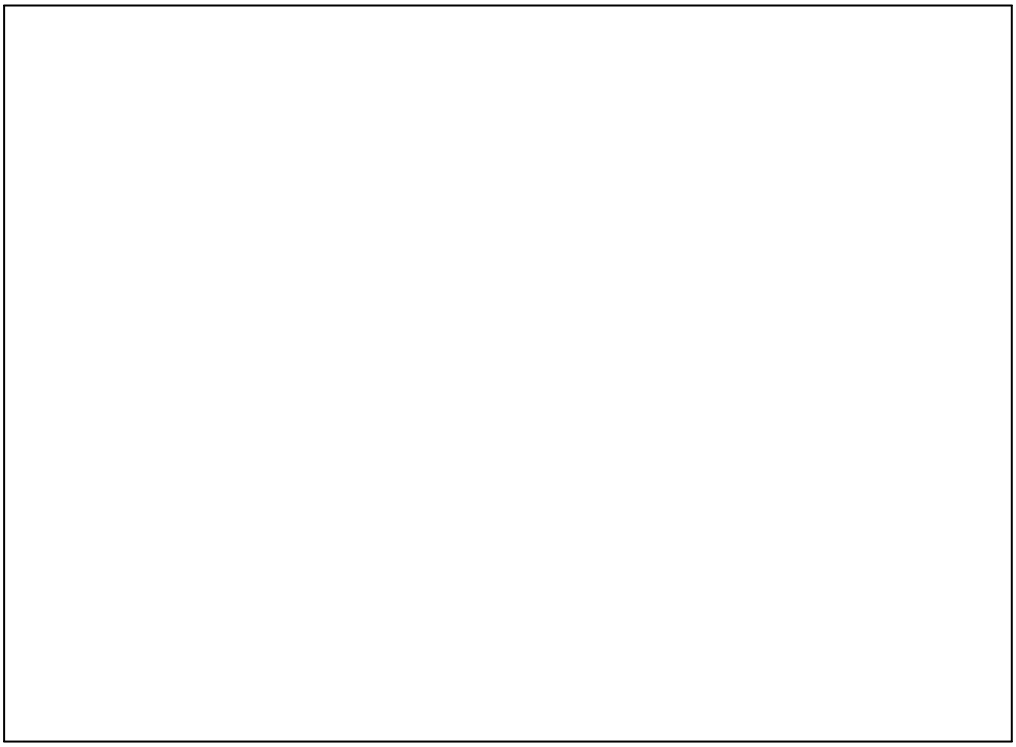
My

Mz

M

Computation of the compressive force in k-Newton/meter=N/mm on the wire at the coil edge (radial unit: 0.25 mm):

$liter := 1 .. 50$ $\quad$ $farad_{liter} := My_{liter, 40} \cdot I0$



$farad_{liter}$

liter

Computation of the axial force acting on the different layers:

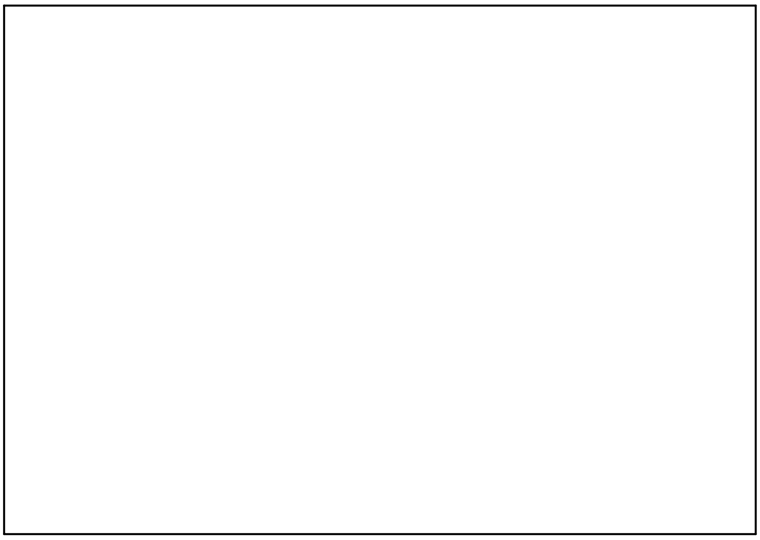
$m := 1 .. 20 \qquad r := 6, 8 .. 28$

$—Í_m =$

$$—Í_m := \sum_{r=1}^{12} My_{4+2\cdot r,\, m\cdot 2} \cdot I0 \cdot a_r \cdot 2 \cdot \pi$$

: force produced by layer in position z=0.25*m on the total of coil

: Newton



$—Í_m$

m

Summation of all forces within one coil-half side:

$$sum := \sum_{n=1}^{20} —Í_n \qquad sum =$$

:Newton , total force of one coil-half

Computation of the inductance of the coil constructed by several layers of windings:
Length measurements in [mm], Induktivität in [Henry].
Height of the coil la [mm]
Radii of windings a1, a2 to be considered for cross inductance
Diameter of wire d [mm]

la = ∎ d = ∎

Number of windings per layer: la/d

$$L(a,b,L,d) := \frac{2\cdot\pi\cdot 10^{-10}\cdot a\cdot b}{d^2}\cdot\int_0^{2\cdot\pi} -\cos(\phi)\cdot\left[\int_{\frac{-L}{2}}^{\frac{L}{2}}\left[\ln\left[\frac{\left(\frac{L}{2}-z\right)^2+\sqrt{\left(\frac{L}{2}-z\right)^2+a^2+b^2+2\cdot a\cdot b\cdot\cos(\phi)+10^{-5}}}{\sqrt{a^2+b^2+2\cdot a\cdot b\cdot\cos(\phi)+10^{-5}}}\right] \ldots + \ln\left[\frac{\left(\frac{L}{2}+z\right)^2+\sqrt{\left(\frac{L}{2}+z\right)^2+a^2+b^2+2\cdot a\cdot b\cdot\cos(\phi)+10^{-5}}}{\sqrt{a^2+b^2+2\cdot a\cdot b\cdot\cos(\phi)+10^{-5}}}\right]\right] dz\right]$$

test result of present calculation: $L(5,5,1000,1) =$ ∎ ***Henry***

classical result for "long" coil of one layer and 1000 windings $4\cdot\pi\cdot 10^{-7}\cdot 10^{6}\cdot 25\cdot\pi\cdot\frac{10^{-3}}{1000} =$ ∎ ***Henry***

Computation of the inductance of the magnetic coil in consideration

$\text{Lteil}(i,j) := L\left(a_i\cdot 0.8, a_j\cdot 0.8, la, d\right)$ NL = ∎ Lteil(1, 1) = ∎ Lteil(12, 12) = ∎

$$\text{Induk} := \sum_{i=1}^{NL}\sum_{j=1}^{NL}\text{Lteil}(i,j)$$

Induk = ∎ ***:Henry***

$m := 1 .. NL$ $n := 1 .. NL$

$ML_{m,n} := \text{Lteil}(m,n)$

$ML = ■$

Fixing of the system parameters:

$coul := C2$ $coul = ■$ ***[Farad] capacity of the bank***

$\rho := 1.95 \cdot 10^{-6}$ ***Ohm*cm at T=273 K*** $Lges := Induk$

$\tau := \pi \cdot 1000 \cdot \sqrt{Lges \cdot coul}$ $\tau = ■$ ***[msec] pulse length***

$\mu 0 := 4 \cdot \pi \cdot 10^{-9}$ ***V*sec/A*cm***

Skin depth in wire:: $\delta RT := \sqrt{0.2 \cdot \tau \cdot \frac{\rho}{\mu 0 \cdot \pi}}$ $\delta RT = ■$ ***[mm] Skin depth at T=273 K***

$Rblind := \sqrt{\frac{Lges}{coul}}$ $Rblind = ■$ ***[Ohm] ratio of U/I***

$Umax := Rblind \cdot I0$ $Umax = ■$ ***[kV] Voltage for maximum field***

$\delta 77 := \delta RT \cdot \sqrt{0.13}$ $\delta 77 = ■$ ***[mm] skin depth at T=77 K***

$\delta He := \delta RT \cdot \sqrt{0.04}$ $\delta He = ■$ ***[mm] skin depth T=4.2 K***

Calculation of temperature increase in coil after shot considering the "Action Integral" for a half-sinus current pulse of length τ and maximal current density j:
0.5*τ*j^2=Integral from starting temperature Tex before the shot to final temperature Tf after pulse with length τ of integrand [ρd*c(T)/ρCu]
ρd: density of Cu (not temperature dependent)
c(T) specific heat as function of temperature T

ρCu(T) specific resistance as function of temperature T

Temperature dependence of the specific resistance of Cu:
Data after HENNING (Fritz Herlach)

Resistance values:

$r_{16} := 1.0$ $r_{15} := 0.92$ $r_{14} := 0.68$ $r_{13} := 0.46$ $r_{12} := 0.23$ $r_{11} := 0.15$ $r_{10} := 0.065$

$tesla_{16} := 273.0$ $tesla_{15} := 250.0$ $tesla_{14} := 200.0$ $tesla_{13} := 150.0$ $tesla_{12} := 100.0$ $tesla_{11} := 80.0$ $tesla_{10} := 60.0$

$r_9 := 0.0195$ $r_8 := 0.014$ $r_7 := 0.0097$ $r_6 := 0.0074$ $r_5 := 0.0063$ $r_4 := 0.0057$ $r_3 := 0.005$

$tesla_9 := 40.0$ $tesla_8 := 35.0$ $tesla_7 := 30.0$ $tesla_6 := 25.0$ $tesla_5 := 20.0$ $tesla_4 := 15.0$ $tesla_3 := 10.0$

$r_2 := 0.0048$ $r_1 := 0.0044$ $r_0 := 0.004$

$tesla_2 := 8.0$ $tesla_1 := 6.0$ $tesla_0 := 4.0$

$vs := cspline(tesla, r)$ $\rho cu(T0) := interp(vs, tesla, r, T0) \cdot \rho$ ***[Ohm*cm]***

Data after LANDOLT-BÖRNSTEIN:

$q_0 := 0.0005$ $Tq_0 := 4$ $q_1 := 0.00016$ $Tq_1 := 14.558$ $q_2 := 0.00157$ $Tq_2 := 23.278$

$q_3 := 0.00638$ $Tq_3 := 30.972$ $q_4 := 0.01380$ $Tq_4 := 36.80$ $q_5 := 0.02744$ $Tq_5 := 43.162$

$q_6 := 0.04516$ $Tq_6 := 49.032$ $q_7 := 0.08050$ $Tq_7 := 57.528$ $q_8 := 0.12628$ $Tq_8 := 66.449$

$q_9 := 0.16937$ $Tq_9 := 73.68$ $q_{10} := 0.24208$ $Tq_{10} := 84.921$ $q_{11} := 0.33307$ $Tq_{11} := 98.169$

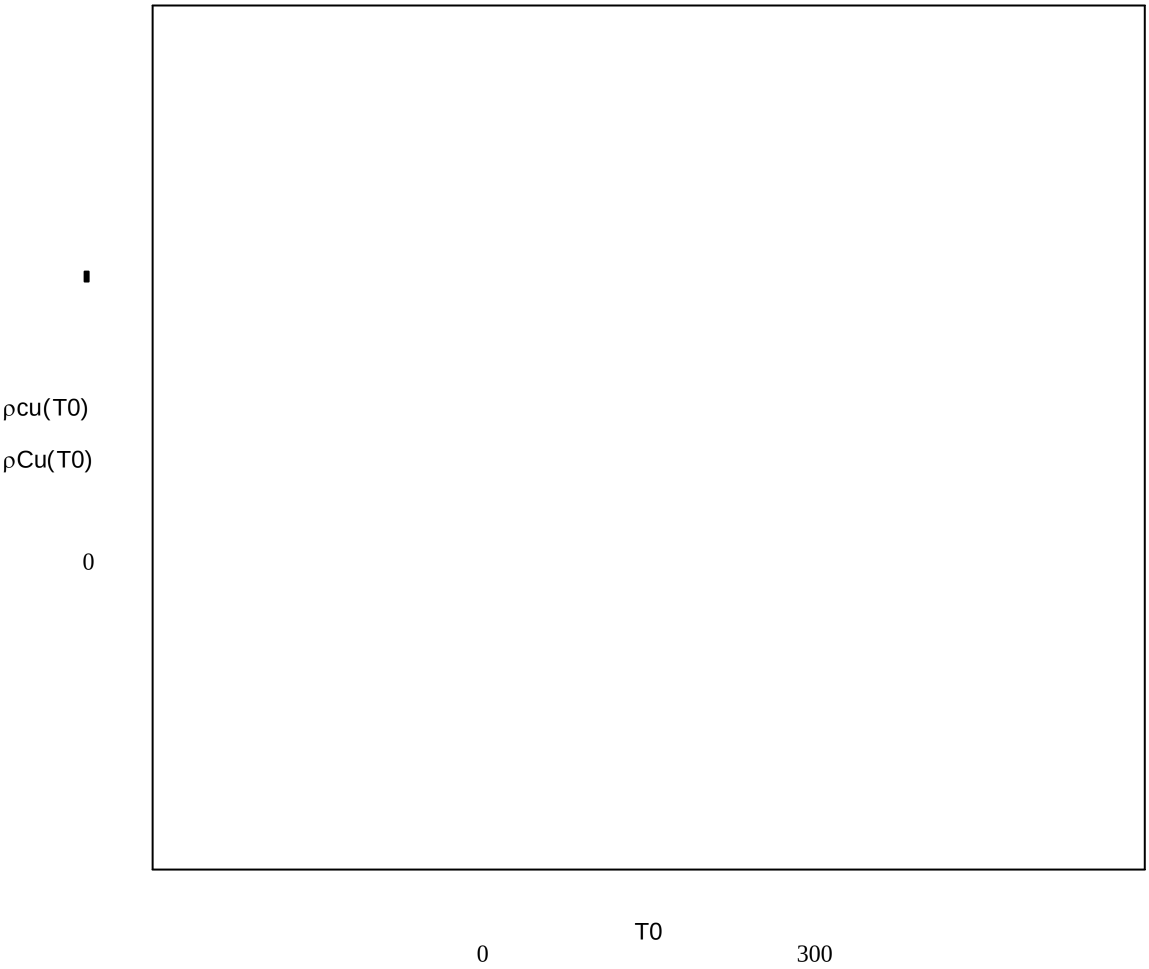
$q_{12} := 0.42132$ $Tq_{12} := 110.620$ $q_{13} := 0.46746$ $Tq_{13} := 117.178$ $q_{14} := 0.58023$ $Tq_{14} := 133.033$

$q_{15} := 0.7191$ $Tq_{15} := 152.720$ $q_{16} := 0.8784$ $Tq_{16} := 175.55$ $q_{17} := 0.9687$ $Tq_{17} := 188.174$

$q_{18} := 1.1017$ $Tq_{18} := 208.061$ $q_{19} := 1.3865$ $Tq_{19} := 250.187$ $q_{20} := 1.7055$ $Tq_{20} := 297.855$

$vsq := cspline(Tq, q)$ $\rho Cu(T0) := interp(vsq, Tq, q, T0) \cdot 10^{-6}$ ***[Ohm*cm]***

$T0 := 4, 5 .. 300$



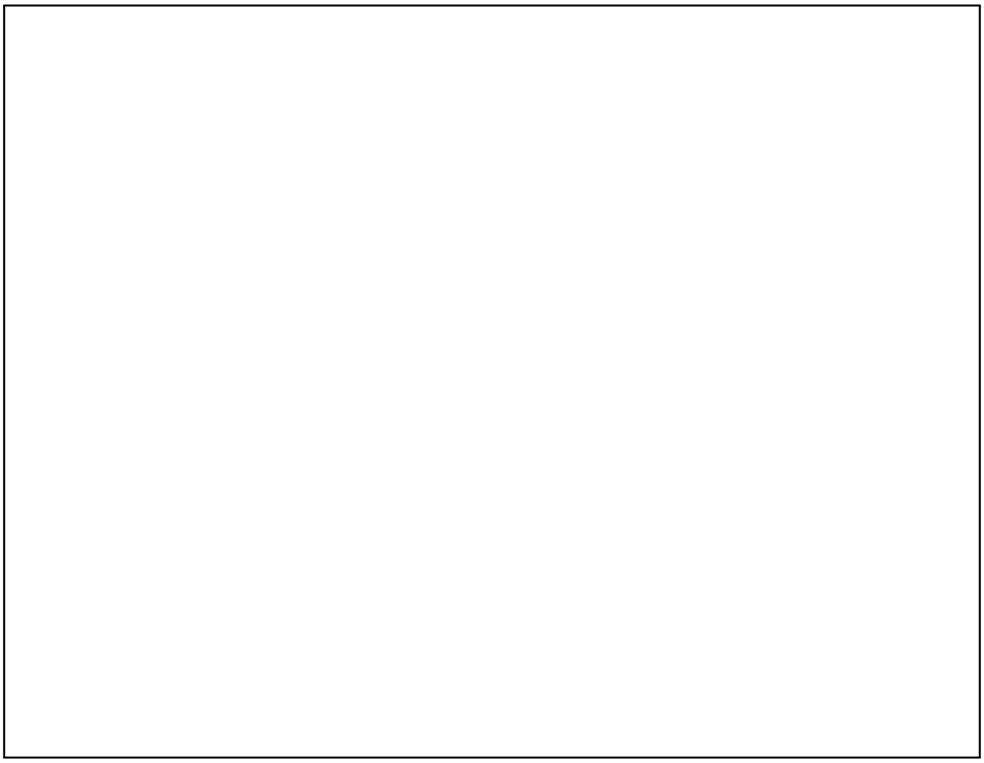
: Ohm*cm

Specific Heat after Debye:

$$c(T0) := \left(\frac{T0}{343}\right)^3 \cdot \left[\int_0^{\frac{343}{T0}} \frac{x^4 \cdot e^x}{\left(e^x - 1\right)^2}\,dx\right] \cdot 1.2017$$

$c(4.2) =$ 　　 $c(77) =$

$T0 := 5, 6 .. 250$



$Tex := 4.2$ ***[K] starting temperature of the coil before shot***

$j = ▪$ ***:maximum current dernsity***

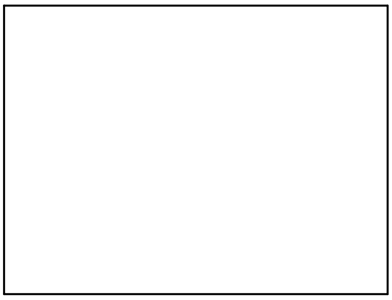
$$w(T0) := \int_{Tex}^{T0} \frac{c(tesla)}{\rho Cu(tesla)}\, dtesla$$

$$f := \frac{0.5\cdot\tau\cdot j^{2}\cdot 1.6501}{\rho d}\cdot 10^{6}$$

:Factor 1.6501 resulting from magneto resistance of the form: (1+0.00766*B(t)) [after Fritz HERLACH]

$f = ▪$

$T0 := 10, 40 .. 500$



$T0 := 7$ $\qquad Tf := root\left(\frac{w(T0)}{f} - 1, T0\right)$ $\qquad Tf = $ ■ ***[K] final temperature of the coil after shot***

$Tex := 77$ ***[K] starting temperature of the coil before shot***

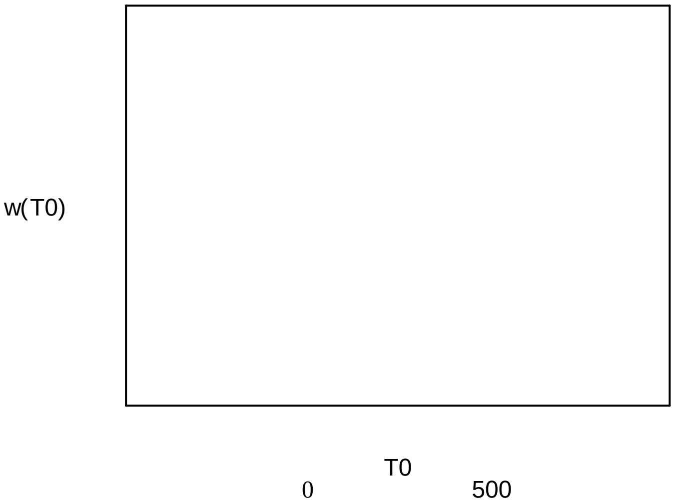
$j = $ ■ ***:maximum current density***

$$w(T0) := \int_{Tex}^{T0} \frac{c(tesla)}{\rho Cu(tesla)}\, dtesla \qquad f := \frac{0.5 \cdot \tau \cdot j^{2} \cdot 1.6501}{\rho d} \cdot 10^{6}$$

:Factor 1.6501 resulting from magneto resistance of the form: (1+0.00766*B(t))

$f = $ ■

$T0 := 10, 40 .. 500$



$T0 := 100$ $\qquad Tf := root\left(\frac{w(T0)}{f} - 1, T0\right)$ $\qquad Tf = $ ■ ***[K] final temperature of the coil after shot***

$d\phi$